GCSE Maths – Algebra

Straight Line Graphs

Notes

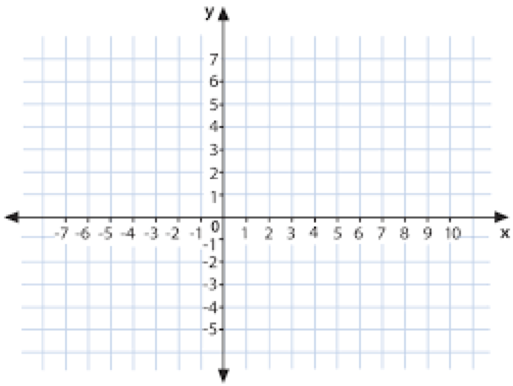
WORKSHEET

Graphs

Quadrants

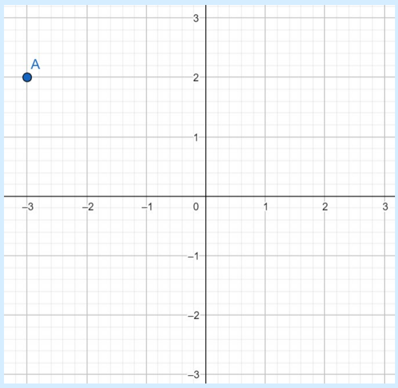
When we extend the x and y axes beyond the origin, we obtain **negative axes** in addition to the **positive axes**. The four areas divide by the axes are called **quadrants**.

- When the x coordinate is to the **left** side of the origin, the number is **negative**.
- When the y coordinate is **under** the origin, the number is **negative**.

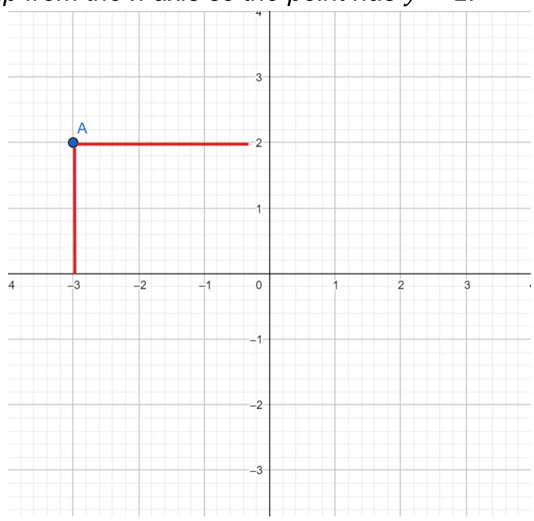


To plot points, the rule **"across the corridor and up or down the stairs"** applies. We consider the x-coordinate followed by the y-coordinate.

Example: State the coordinates of point A.



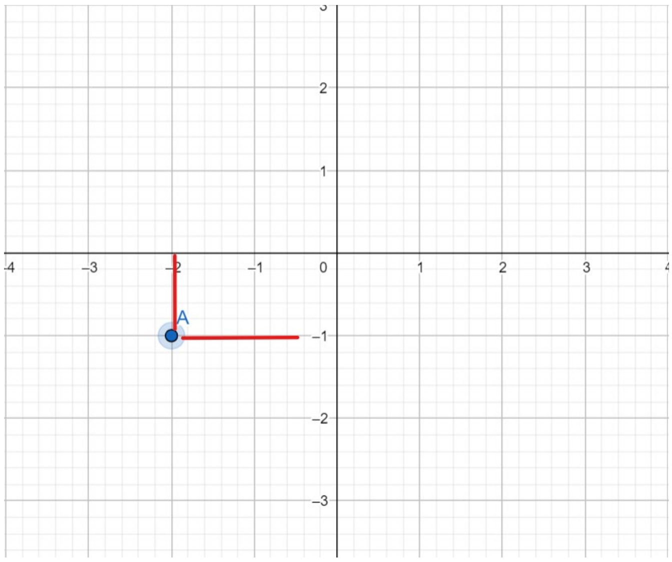
Point A is to the left of the y-axis by 3 *units. This means the coordinate has* $x = -3$.
The point is 2 *units up from the x-axis so the point has* $y = 2$.



The coordinates for point A are $(-3, 2)$.

Example: Plot the coordinates (-2, -1)

The x value is -2. This means the point is 2 units from the y-axis in the negative horizontal direction, i.e. the point is 2 units left of the origin.
The y value is -1. This means the point is 1 unit from the x-axis in the negative vertical direction, i.e. the point is 1 unit down from the origin.



Equation of a straight line

A **straight-line graph** has the general equation

$$y = mx + c.$$

In the equation:

- m is a constant that represents the **gradient** of the graph. The gradient is sometimes referred to as the slope and represents the steepness of the graph.
- c is the **y-intercept** which represents the point where the line crosses the y-axis.

In order to find the equation of a straight line, the **gradient** and **y-intercept** must be known.

When a point on the line is given and the gradient is known, we can find the equation by **substituting** the values in the equation $y = mx + c$. This will determine the value of c and the equation of the line can be found.

Calculating gradient

The gradient of a line can be viewed as the **change in y** over the **change in x**. Therefore, the gradient of a line can calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) represent the coordinates of **two points** on the line.

Calculating the equation of a line

When **two points** on a line are known, the **gradient** can be calculated using the formula above. Then, by **substituting** in a coordinate on the line, the **y-intercept** c can be found, meaning the full equation of the line can be obtained.

Example: Line L has a gradient equal to -1 and passes through the point $(3,1)$. Find the equation of Line L.

1. Find the value of the y-intercept c by substituting the value for m and the coordinates of the point on the line into the general equation into $y = mx + c$.

$$\boldsymbol{y = mx + c}$$

$$1 = -1(3) + c$$
$$1 = -3 + c$$
$$\boldsymbol{c = 4}$$

2. Substitute the values for m and c into the general form $y = mx + c$ to obtain the equation of the line.

   Since $m = -1$ and $c = 4$, substituting into $\boldsymbol{y = mx + c}$ we have

$$\boldsymbol{y = -x + 4.}$$

Example: Line M passes through the points $(-1, -4)$ and $(2, 11)$. Find the equation of Line M in the from $y = mx + c$.

1. Using the formula, calculate the gradient of Line M.

   We use the two points $(x_1, y_1) = (-1, -4)$ and $(x_2, y_2) = (2,11)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - (-4)}{2 - (-1)} = \frac{15}{3} = 5$$

2. Find the value of the y-intercept c by substituting the value for m and the coordinates of the point on the line into the general equation into $y = mx + c$.

$$\boldsymbol{y = mx + c}$$

$$-4 = 5(-1) + c$$
$$-4 = -5 + c$$
$$\boldsymbol{c = 1}$$

3. Substitute the values for m and c into the general form $y = mx + c$ to obtain the equation of the line.

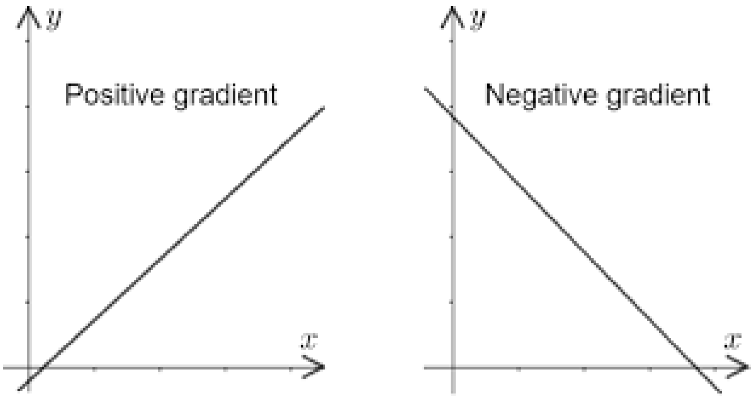
   Since $m = 5$ and $c = 1$, substituting into $\boldsymbol{y = mx + c}$ we have

$$\boldsymbol{y = 5x + 1.}$$

Finding the gradient graphically

The gradient measures the **steepness** of the graph. The gradient can be interpreted by looking at the graphs:

- Graphs that slope **up** from **left to right** have a **positive** gradient.
- Graphs that slope **down** from **left to right** have a **negative** gradient.

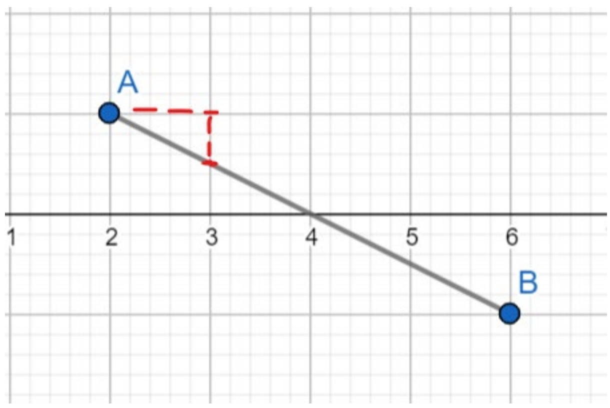


The **numerical** value of the gradient can also be interpreted from the line:

For every unit the x value moves **across one** to the right, the **y value changes**: the amount the y value changes by is the value of the gradient.

*For example, if for every one unit across in the positive x-direction, the y value **decreases** by 2, the gradient would be -2.*

Example: Find the gradient of the line below



For every one unit across in the positive x-direction, the y value decreases by $\frac{1}{2}$ a unit. Since the line slopes down from left to right, the gradient must be negative so the numerical value of the gradient must be $-\frac{1}{2}$.

Plotting straight line graphs

There are various ways to draw a graph when given an equation of a straight line:

- We can use the **y-intercept** and **gradient** to plot the graph. The point c representing the y-intercept means we plot the point $(0, c)$. We then use the gradient to plot new points on the line: we move one unit in the positive x-direction and m units in the vertical y-direction and plot the new position arrived at. Repeating this, we obtain the line by connecting these points with a straight line.

- We can calculate a **table of values** which will give us points to plot. Connecting the points with a straight line will give us the required line. To draw the graph, we **substitute** x values into the equation, to calculate the y values. Then, we **plot** the coordinates onto the graphs and **join** them up with a straight line.

Example: Draw the graph $y = 3x + 5$ between $-2 \leq x \leq 2$

1. Create a table with x values between -2 and 2.

x	-2	-1	0	1	2
y					

2. Calculate the y values by substituting the x values into the equation $y = 3x + 5$.

x	-2	-1	0	1	2
y	-1	2	5	8	11

For example, the value of y corresponding to $x = -2$ is obtained by:

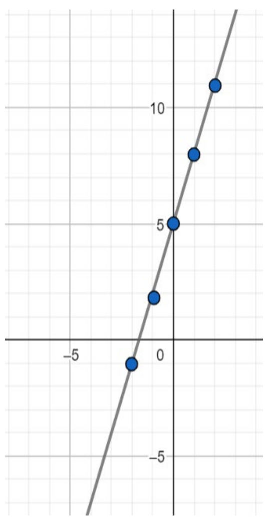
$$y = 3x + 5$$

$$y = 3(-2) + 5$$

$$y = -6 + 5 = \mathbf{-1}$$

3. Plot these points on the graphs and join them up with a straight line.

We plot the points $(-2, -1)$, $(-1, 2)$, $(0, 5)$, $(1, 8)$ and $(2, 11)$:



Parallel and perpendicular lines

Parallel lines

Parallel lines are lines which are always the same distance apart, meaning parallel lines never meet. Lines can be identified as being parallel if they have the **same gradient**. This means that the value of **m is the same** in the equations of parallel lines.

Perpendicular lines

When two lines meet at a **right-angle**, they are **perpendicular**. For example, the lines which join a corner of a square are perpendicular.

If two lines are perpendicular, then the **product** of the two lines' gradients is **-1.**

This means that if we have the equation of one line with gradient m, the line perpendicular to this line has gradient equal to $-\frac{1}{m}$. This gradient is the **negative reciprocal** of the first lines' gradient.

Example: Determine whether the lines $y = -2x + 2$ and $2x + 2y = 6$ are parallel.

1. Identify the condition which must be checked to identify parallel lines.

   If the two the lines are parallel, the gradient m *will be the same in both equations when put into the general form* $y = mx + c$.

2. Find the gradient of the first line $y = -2x + 2$.

   The first line $y = -2x + 2$ *is already in the form* $y = mx + c$ *so we can see that it has gradient* $\mathbf{m} = -\mathbf{2}$.

3. Rearrange the second line $2x + 2y = 6$ into the general form $y = mx + c$ to find the gradient of the second line.

   We need to rearrange the second line $2x + 2y = 6$ *into the general form* $y = mx + c$*:*

$$2x + 2y = 6$$
$$2y = -2x + 6$$
$$y = -x + 3$$

   Comparing this line to the form $y = mx + c$*, we see it has gradient* $\mathbf{m} = -\mathbf{1}$.

4. Compare the gradients to determine if the lines are parallel.

   The gradients for the two lines are not equal and therefore the lines are ***not parallel***.

Example: Find the equation of the line that is parallel to $y = 3x + 1$ and passes through the point $(-1, -5)$.

1. Use the property of parallel lines to find the gradient of the new line.

   Since the two lines are parallel, the gradient of the new line must be equal to the gradient of the line $y = 3x + 1$.

   Therefore, the gradient of the new line must be $m = 3$.

2. Find the value of the y-intercept c by substituting the value for m and the coordinates of the point on the line into the general equation into $y = mx + c$.

$$\boldsymbol{y = 3x + c}$$

$$-5 = 3(-1) + c$$
$$-5 = -3 + c$$
$$\boldsymbol{c = -2}$$

3. Substitute the values for m and c into the general form $y = mx + c$ to obtain the equation of the line.

   Since $m = 3$ *and* $c = -2$, *substituting into* $\boldsymbol{y = mx + c}$ *we have*

$$\boldsymbol{y = 3x - 2}.$$

Example: Determine whether the lines $y = \frac{1}{2}x + 2$ and $2x + y = 6$ are perpendicular.

5. Identify the condition which must be checked to identify perpendicular lines.

   When two lines are perpendicular, the product of two lines' gradients is equal to -1.

6. Find the gradient of the first line $y = \frac{1}{2}x + 2$.

   The gradient of the first line $y = \frac{1}{2}x + 2$ *is* $\boldsymbol{m_1} = \frac{\mathbf{1}}{\mathbf{2}}$.

7. Rearrange the second line $2x + y = 6$ into the general form $y = mx + c$ to find the gradient of the second line.

$$2x + y = 6$$
$$y = -2x + 6$$

   The gradient of the second line is $\boldsymbol{m_2} = \mathbf{-2}$.

8. Calculate the product of the gradients and use the value to determine if the lines are perpendicular.

   The product of the gradients is $\frac{1}{2} \times -2 = -1$.

   The product of the gradients is -1 *and therefore the lines are* ***perpendicular*** *to each other.*

Example: Line A passes through the points $(3, 6)$ and $(5, 12)$. Given that line B is perpendicular to line A and passes through $(0, 3)$, find the equation of line B in the form $y = mx + c$.

1. Use the property of perpendicular lines to find the gradient of line B.

   When two lines are perpendicular, the product of their gradients is -1.

   Using $(x_1, y_1) = (3, 6)$ *and* $(x_2, y_2) = (5, 12)$, *the gradient of line A is*

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 6}{5 - 3} = \frac{6}{2} = 3$$

   So the gradient of line A is $\boldsymbol{m_A = 3}$.

   If m_B *is the gradient of line B, then we must have*

$$\boldsymbol{m_A \times m_B = 3 \times m_B = -1}$$

   Therefore, the gradient of line B must be $m_B = -\frac{1}{m_A} = -\boldsymbol{\frac{1}{3}}$.

2. Find the value of the y-intercept c by substituting the value for m and the coordinates of the point on the line into the general equation into $y = mx + c$.

$$\boldsymbol{y = mx + c}$$

$$3 = -\frac{1}{3}(0) + c$$
$$\boldsymbol{c = 3}$$

3. Substitute the values for m and c into the general form $y = mx + c$ to obtain the equation of the line B.

   Since $m_B = -\frac{1}{3}$ *and* $c = 3$, *substituting into* $\boldsymbol{y = mx + c}$ *we have*

$$\boldsymbol{y = -\frac{1}{3}x + 3}.$$

Straight Line Graphs – Practice Questions

1. Draw the graph $y = 4x - 10$ between $-2 \leq x \leq 2$.

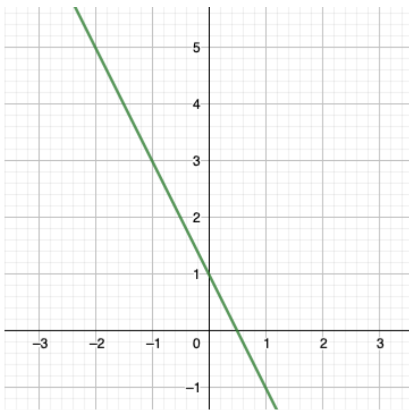
2. The gradient of a line is equal to 2 and passes through the point $(-5,3)$. Find the equation of the line.

3. The gradient of a line is $-\frac{1}{3}$ and passes through the point $(-6,-2)$. Find the equation of the line.

4. A line passes through the points $(2,10)$ and $(1,3)$. Find the equation of the line in the from $y = mx + c$.

5. A line passing through the origin also passes through the point $(1,-7)$. Find the equation of the line in the from $y = mx + c$

6. Find the gradient of the line given below:



7. Determine whether the line $2y = 6x + 3$ and $4x + 6y = 1$ are parallel.

8. Write the equation of the line parallel to $y = -x + 2$ passing through point $(9,6)$.

9. Line A passes through the points $\left(-\frac{1}{2}, 4\right)$ and $(2,9)$. Given that line B is perpendicular to line A and passes through $(-4,-5)$, find the equation of line B in the form $y = mx + c$.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.